HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM TEST (Group 1) Semester 3, Academic year 2018-2019 Duration: 90 minutes

SUBJECT: Calculus 2 Chair of Department of Mathematics Lecturer:

Signature: Signature:

Full name: Full name: Assoc.Prof. Mai Duc Thanh

Instructions:

• Each student is allowed a maximum of two double-sided sheets of reference material (of size A4 or similar). All other documents and electronic devices, except scientific calculators, are not allowed.

• Each question carries 20 marks.

Question 1. Find the following limits:

a) n→∞lim (ln(6n2 + n + 1) − ln(n2 + 2n + 5)) b) n→∞lim n( √ne − 1) Question 2. Determine whether the given series is convergent or divergent:

a)

∑∞n=1

lnnn2 b)

∑∞n=1

sin(n1) Question 3. Find a power series representation for the function f(x) = (1 + x

2x)2 and deter- mine the radius of convergence of the power series.

Question 4. Determine whether the following two lines are parallel, intersecting, or skew. If they are skew, find the distance between them

L1 : x =1+ t, y =1+6t, z = 2t and

L2 : 1+2s, y =5+15s, z = −2+6s Question 5. (a) Find the limit of the given vector function

lim t→0〈√1 + t − t √1 − t

,t2 + 2, 1t − t2 1

+ t〉.

(b) Find parametric equations for the tangent line to the curve r(t) =< tcost, tsint,t > , 0 ≤ t ≤ 2π at the point (0,π/2,π/2).

———END OF QUESTIONS———

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CALCULUS 2 Solutions for Mid-term Test

Question 1. a)

n→∞lim (ln(6n2 + n + 1) − ln(n2 + 2n + 5)) = n→∞lim ln 6nn2 2 + n + + 2n + 1

5 = ln(

n→∞

lim 6+1/n + 1/n2

1+2/n + 5/n2)

= ln 6

b) We have

n→∞lim n( √ne − 1) = x→∞lim x(e1/x − 1)

= x→∞

lim e1/x 1/x − 1

= x→∞

lim e1/x(1/x)

(1/x) = e0 = 1.

Question 2. a) Set

f(x) = lnxx2 , x ≥ 3. Then f(x) is continuous, decreasing, and f(n) = an. Since

∫ ∞3

lnxx2 ∫ ∞dx =

lnxd(−1/x) = ln 3 3/3 − 1/x∣∣∣∞3 = −lnx/x∣∣∣∞3 +

∫ ∞3 = (ln 3 + 1)/3.

1/x2dx

This b) implies We have

the series ∑∞n=3(lnn)/n2 is convergent so is the series ∑∞n=1(lnn)/n2.

The series ∑1/n diverges, by the limit n→∞

lim comparison sin 1/n 1/n

= test, 1

the given series diverges.

Question 3. We have

1 − 1

x =

∑n=0 ∞xn, |x| < 1.

So 1

1+2x = 1 − (−2x) 1

=

∑∞n=0(−2x)n =

∑∞n=0(−2)nxn, |x| < 1/2. Using differentiation

ddx

1 1+2x = −2

(1 + 2x)2 =

∑∞(−2)nnxn−1, n=1|x| < 1/2

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so x

(1 + 2x)2 =

∑∞n=1(−2)n−1nxn, |x| < 1/2. R = 1/2.

Question 4. Let (α) be the plane containing (L1) and parallel to (L2). Then the distance between the skew lines (L1) and (L2) is equal to the distance from M(1,5,−2) on (L2) to (α).

The normal vector n of (α) can be chosen as

n =< 1,6,2 > × < 2,15,6 >=< 6,−2,3 > .

Hence, the plane has the equation6(x − 1) − 2(y − 1) + 3z = 0

or

6x − 2y + 3z − 4=0.

Therefore, the distance is

d = |6 × 1 − √2 6× 2 5+3 × (−2) + 22 + 32 − 4| = 2.

Question 5. (a) It holds that

lim t→0

√1 + t − t √1 − t

= lim t→0

(√1 + t + 2

√1 − t) = 1 lim t→0(1t − t2 t((1 √1 + + t) t 1

+ t− + (1 √1 − − t)

t) = lim t→0 )

= lim t→0

tt(t2 + 2 + t − t) t

= lim t→0

1 t + 1 = 1. So

lim t→0〈√1 + t − t √1 − t

,t2 + 2, 1t − t2 1

+ t〉

=< 1,2,1 >

(b) r(t) =< tcost, tsint, t >, 0 ≤ t ≤ 2π. It holds that

r (t) =< cost − tsint,sint + tcost,1 >, 0 ≤ t ≤ 2π.

The point A(0,π/2,π/2) on the curve corresponds to t = π/2. So r (π/2) =< −π/2,1,1 >. Thus the tangent line has equations

x = −(π/2)t, y = π/2 + t, z = π/2 + t.

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